and of seizing the abundant fishes of the shallow seas in which it lived. Compared with our modern birds, the two features of this ancient form which most forcibly arrest attention are the teeth and the legs. The teeth were covered with smooth enamel, terminating upward in conical pointed crowns and downward in stout fangs, closely resembling those of mosasauroid reptiles. Their mode of growth and replacement have been determined to have taken place in a manner very similar to that in some reptiles, the young tooth forming on the inner side of the fang of the tooth in use, and increasing in size, while a pit for its reception was gradually made by absorption. The old tooth, being progressively undermined, was finally expelled by its successor, the number of teeth thus remaining unchanged. The teeth were implanted in a common alveolar groove, as in *Ichthyosaurus*. In the upper jaw they were confined to the maxillary and entirely absent from the pre-maxillary bone; in the lower jaw they extended from near the anterior extremity of the ramus along the entire upper border of the dentary bone. Mr. Marsh believes that they were held in position by cartilage which permitted some fore and aft movement, but on the decay of which after death the teeth readily became displaced and fell out of the jaw. This is an important fact in its bearing upon the nature of the teeth found on the same slab of Solenhofen limestone with the well-known *Archæopteryx*. These teeth, it will be remembered, were referred by Mr. Evans to the bird itself—a reference fully confirmed by Mr. Marsh, who says that he at once identified the teeth as those of birds and not of fishes, and by the subsequent discovery of other remains of the bird. In *Hesperornis regalis* there appear to have been fourteen functional teeth in the maxillary bone and thirty-three teeth in the corresponding ramus of the lower jaw. The wings are rudimentary or aborted, a remnant of the humerus alone existing. They may have gradually diminished from disuse until, as the power of flight ceased, the legs and feet increased in proportion, and assumed the massive dimensions shown in the specimens, or, as Mr. Marsh suggests, the bird may have been a carnivorous aquatic ostrich, never having possessed the power of flight, but descended from a reptilian ancestry which is strongly recalled by different portions of the skeleton. Among recent birds, the peculiar legs and feet of *Hesperornis* find their nearest analogues in the Grebes of the genus *Podiceps*. They were admirably adapted for propulsion in water, but scarcely served for walking on land. Locomotion must have been entirely performed by the posterior limbs—a peculiarity which distinguishes *Hesperornis* from all other birds recent or fossil. The tail appears to have been composed of twelve vertebræ, unique in their peculiar widely extended transverse processes and depressed horizontal ploughshare bone. Broad and flat, somewhat like that of the beaver, it must have been a powerful instrument in steering the bird through the water.

The second part is devoted to a description of the remains which have been found of birds belonging to a second order of Odontornithes, termed *Odontotormæ*. Unlike *Hesperornis*, they seem to have been all of comparatively small size and to have possessed powerful wings, but very small legs and feet. From that contemporaneous form, and from all other known birds recent and fossil, they are distinguished by certain types of structure which point back to a very lowly ancestry, lower even than the reptile. Their bones, being mostly air-filled, would enable the carcases to float on water until, by decay or the rapacity of other animals, they were separated and dispersed. Hence skeletons of these flying birds are less entire than those of the massive-boned *Hesperornis*. Nevertheless the remains of no fewer than seventy-seven different individuals have been disinterred. These are included in two well-marked genera, *Ichthyornis* and *Apatornis*, and were all small birds, reminding us by their strong wings and delicate legs and feet of the Terns, like which they were probably also aquatic in habit. Besides the reptilian skull and teeth, the birds of this second order were marked by the character of their vertebræ, which in their biconcave structure recall those of fishes. This is the more remarkable, as in *Hesperornis* the vertebræ are like those of modern birds. Yet these two utterly dissimilar types were contemporaries, and their remains have been preserved in the same strata. Mr. Marsh points out that the transition between the two vertebral types may be traced even in the skeleton of *Ichthyornis* itself, where the third cervical vertebra presents a modification in which the ordinary avian saddle-shaped form appears as it were in the act of development from the biconcave ichthyic form.

In a concluding chapter the author briefly touches upon some of the broader biological questions suggested by the structure of the animals described in the volume. The striking differences between the three orders into which Prof. Marsh divides toothed birds—*Archæopteryx*, *Hesperornis*, and *Ichthyornis*—serve to indicate the high antiquity of the class, and to encourage the search for ornithic remains in the earlier Secondary, if not in the later Palæozoic, rocks. The peculiar character of each of the orders Prof. Marsh believes to have been united in some earlier type, of which no trace has yet been found. This ancestral type may have been almost as much a reptile as a bird. The earliest birds were doubtless closely related to the Dinosaurs and Pterodactyles.

Of the plates, thirty-four in number, which accompany and adorn the monograph it is impossible to speak in terms of too great praise. They are strictly and rigidly scientific diagrams, wherein every bone and part of a bone is made to stand out so clearly that it would not be difficult to mould a good model of the skeleton from the plates alone. And yet with this faithfulness to the chief aim of the illustrations there is combined an artistic finish which has made each plate a kind of finished picture. We heartily congratulate the genial Professor of Palæontology at Yale on the advent of this truly imperial volume. May it be the earnest of many more from the rich store of materials which he has so courageously and enthusiastically gathered among the wilds of the far West!

THE THEORY OF DETERMINANTS

A Treatise on the Theory of Determinants and their Applications in Analysis and Geometry. By Robert Forsyth Scott, M.A. (Cambridge: At the University Press.)

THE list of English text-books on the subject of Determinants is comparatively meagre, and this notwithstanding the fact that the first separate treatise of all

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on the subject was the work of an Englishman, now the distinguished president of the Royal Society. Dr. Spottiswoode's "Elementary Theorems Relating to Determinants" appeared in 1851 (4to, pp. viii. + 63, London, Longmans), and, as a pioneer work, was eminently successful; at the honouring request of the editor of *Crelle* it was republished, with additions, in that well-known journal three or four years later (vol. li. pp. 209-271, 328-381). After considerable intervals came Dodgson's "Elementary Treatise" (London, Macmillan, 1867) and a pamphlet by Wright; and here, until quite recently, the list ended. The chapters on the subject by Todhunter and others belong to a different category, but deserve to be mentioned, as it is doubtless in part owing to their existence that separate treatises have been so rare.

In view of the dearth referred to, he would be a very captious critic indeed who would not gladly welcome the handsome volume whose title is given at the head of this notice. In form and general outward appearance it resembles Part I. of Thomson and Tait's "Elements of Natural Philosophy," and extends to about 250 pages. The matter is arranged under fourteen chapters, the first seven being meant to deal with determinants in themselves, the last seven with the so-called applications; the line of separation, however, is not very well maintained.

In the introductory chapter we have the usual account of permutations, inversions of order, &c., and the usual definition of a determinant; but this is followed by something less familiar, viz., a page or two of exposition regarding Grassmann's "alternate units" or "polar elements," and by the establishment of the theorem that a determinant is expressible as a product of alternate numbers linear in the elements.[1] The constant use afterwards made of this theorem—if theorem we can call that which is but a symbolical expression of the ordinary definition of a determinant—is the distinguishing feature of Mr. Scott's mode of treating the subject. There may be room for doubt whether the study of determinants is thus, as he says, much simplified—the example of § 20, p. 15, is not a happy introductory instance of such simplification—and it may certainly be questioned whether beginners should have the subject at first presented to them in this way; but undeniably a freshness is thereby imparted to the book, which will make it pleasant reading to those who already know something of the matters in hand. Chapters II. to V., on "General Properties of Determinants," "The Minors and the Expansion of a Determinant," "Multiplication of Determinants," and "Determinants of Compound Systems," contain proofs and illustrations of *most* of the well-known general theorems. One might, however, fairly expect so large a work as the present to be more complete in this respect; the omissions for example, of Sylvester's beautiful theorem expressing the product of two determinants as a sum of like products is not easily excusable. Chapter VI., on "Determinants of Special Forms," is good, and the same may be said of the next, which treats of "Determinants with Multiple Suffixes." Belonging to the first part, although included under "Applications," are Chapters IX. and XII. The one concerns what are called "Rational Functional Determinants," but which might be more fitly designated as "Alternants"—to use one of Sylvester's happiest coinages; the other concerns "Determinants of Functions of the same Variable," a title again which is anything but sufficiently discriminative. Both chapters are fresh and interesting. To the borderland between the two parts may be assigned Chapter X., on "Jacobians and Hessians;" then there are chapters (VIII., XI., XIII.) dealing with the applications to three departments of Analysis, viz., Theory of Equations, Theory of Quadrics, and Continued Fractions; and, lastly, there is a very readable chapter (XIV.) on "The Applications to Geometry."

No exercises for the student are given under the individual chapters, but a considerable collection is placed towards the end.

Following this is a "List of Memoirs and Works Relating to Determinants," the arrangement being alphabetical according to authors' names. Mr. Scott acknowledges the incompleteness of the list; but, all the same, one cannot help expressing regret that such an excellent opportunity of publishing an exhaustive, or tolerably exhaustive, bibliography of determinants was lost. Had the list been a judicious *selection*, there would have been less cause for regret, but not rarely the worthless are taken and the good left out. Cauchy, who in a sense laid the foundations of the whole subject, is not once mentioned; Grassmann, who laid the foundation of Mr. Scott's method, is not included; Nägelsbach's name occurs, but his most important paper is omitted: and many more such instances might be cited.

Mr. Scott has given us a very acceptable addition to our mathematical text-books: a little more of the conscientious labour he has shown would have produced a work still more worthy of the press which has issued it.

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*]

[*The Editor urgently requests correspondents to keep their letters as short as possible. The pressure on his space is so great that it is impossible otherwise to ensure the appearance even of communications containing interesting and novel facts.*]

The Stone in the Nest of the Swallow

THE name of swallow's stone was preserved in France even to our days, for Dr. Patrin, a member of the French Institute and of the Academy of Sciences in St. Petersburg, wrote in the "Dictionary of Natural History," Paris, Deterville, 1803 (V° Agate), as follows:—

"On trouve dans les ruisseaux des environs de Sassenage en Dauphiné de très petites Calcedoines ou Agates de forme lenticulaire qu'on a nommées *pierres de Chélidoine*, parce qu'elles ont quelque ressemblance avec les semences de cette plante, *pierres d'hirondelle* parcequ'on en a trouvé dans l'éstomac de ces oiseaux."

Of course this naturalist did not try to throw light on the legend, or to explain the confusion made by some authors between the respective skill of the eagles in geology, and of the swallows in botany, which Phile, in his "Remedies Against Sortileges," clearly sets out in the following verses:

φθορᾶς δὲ τηρεῖ τὰς γονὰς ὑπερτέρας
εἰς τὴν καλίαν ἀετὸς κρύψας λίθον
ὡς ἡ χελιδὼν τοῦ σελίνου τὴν κόμην
Ἐξάψασα δὲ τοῦ τραχήλου τὸν λίθον
κυοῦσα γυνὴ κερδαινεῖ ζῶν τὄβρεφος.

"A stone which the eagle conceals in her nest (aery) preserves

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[1] *E.g.* $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = (a\epsilon_1 + b\epsilon_2 + c\epsilon_3)(d\epsilon_1 + e\epsilon_2 + f\epsilon_3)(g\epsilon_1 + h\epsilon_2 + k\epsilon_3)$, where $\epsilon_1\epsilon_2\epsilon_3 = 1$. and ϵ_1, ϵ_2, ϵ_3 are symbols subject to the laws of ordinary algebra, except that $\epsilon_r\epsilon_s = -\epsilon_s\epsilon_r$, and therefore $\epsilon_r^2 = 0$.

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